

Pwe 1 Ex 6D Q10 (SBD)

$$x^2 - 8x + y^2 - 14y + 56 = 0$$

Substitute $y = kx$ to find intersections;

$$x^2 - 8x + (kx)^2 - 14(kx) + 56 = 0$$

Collect terms:

$$(1 + k^2)x^2 + (-8 - 14k)x + 56 = 0$$

We need values of k for which this has two solutions

ie. Discriminant: $b^2 - 4ac > 0$

$$\text{so } (-8 - 14k)^2 - 4 \times (1 + k^2) \times 56 > 0$$

$$196k^2 + 224k + 64 - 224k^2 - 224 > 0$$

Collect terms:

$$-28k^2 + 224k - 160 > 0$$

$$(x-1) \quad \underline{\underline{28k^2 - 224k + 160 < 0}} \quad \text{gives two points of intersection}$$

$$\text{Roots: } k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = 0.7928... \text{ and } 7.207...$$

∴ Two intersections when $0.79 < k < 7.21$