Pure 1 Ex6F Q6 (SBD)

a) $AB^2 = (8-1)^2 + (4--3)^2 = 98$

$BC^2 = (1--4)^2 + (-3-2)^2 = 50$

$AC^2 = (8--4)^2 + (4-2)^2 = 148$

So $AC^2 = AB^2 + BC^2$, this is only true when the triangle is right-angled.

Alternative method

gradient of $AB = \frac{4-3}{8-1} = 1$

gradient of $BC = \frac{-3-2}{1--4} = -1$

As gradient of $BC = \frac{-1}{\text{gradient of } AC}$, they must be perpendicular - i.e. they meet at a right-angle, therefore $\triangle ABC$ is right-angled.

b) As $\angle ABC = 90°$, AC must be a diameter of the circle

∴ Centre of the circle is the midpoint of AC: (2, 3)

and radius $= \frac{1}{2}AC = \frac{1}{2}\sqrt{148} = \sqrt{37}$

So circle equation is: $\underline{(x-2)^2 + (y-3)^2 = 37}$

c) D(3, 9) - sub in eqn: $(3-2)^2 + (9-3)^2 = 36+1 = 37$

Therefore D is on the circle circumference