

Pure 1 Ex 6F Q8 (SBD)

$\angle DEF = 90^\circ \therefore \triangle DEF$ is a right-angled triangle

$$\text{So } DE^2 + EF^2 = DF^2$$

$$\text{But } DE^2 = 4^2 + (b-1)^2$$

$$EF^2 = 4^2 + (b-7)^2$$

$$DF^2 = 8^2 + 6^2$$

$$\therefore 4^2 + (b-1)^2 + 4^2 + (b-7)^2 = 100$$

$$16 + b^2 - 2b + 1 + 16 + b^2 - 14b + 49 = 100$$

so, collecting terms

$$2b^2 - 16b - 18 = 0$$

$$b^2 - 8b - 9 = 0$$

$$(b-9)(b+1) = 0$$

But $b > 0$, so $b = 9$

b) As $\angle DEF = 90^\circ \Rightarrow DF$ is a diameter of the circle

\therefore Centre of the circle is their midpoint $\therefore (-3, 4)$

and radius is $\frac{1}{2}DF = \frac{1}{2}\sqrt{100} = 5$

\therefore C's equation is $(x+3)^2 + (y-4)^2 = 25$